[13.24] Show 

Let A = and  = 

Det() = 

= 

I provide 2 proofs, one using tensor symbols and one using diagrammatic notation.

**Tensor Proof:** Det() =



This is a double sum over permutations of (*a*,*b*,…,*j*) and (*r*,*s*,…,*z*). Fix a permutation of (*a*,*b*,…,*j*). The only non-zero term is



(Note: The above is actually multiplied by the permutation of . However, this factor can be omitted because all such products are equal to  as I showed in the proof of Problem [13.22].)

There are *n*! permutations of (*a*,*b*,…,*j*), so

Det() =  =  ✔

**Diagrammatic Proof:** Det() = 

= 

+ ∈ + + + o(∈2) 

=   

+ ∈    + +  

 + ∈ + +

  *n*! + ∈ (n – 1)! Tr + … + (*n* – 1)! Tr

=  [ *n*! + ∈ (n) (*n* – 1)! Tr(A) ]

= 1 + ∈ Tr(A)

*c*

*b*

*a*

*t*

*s*

*r*

(\*) Let B = .

Let P*ab…c* and P*rs…t* be the sets of permutations of (*a*,*b*,…,*c*) and (*r*,*s*,…,*t*), respectively.

B = .

Fix **. The only non-zero term in the sum is

.

(Note: I showed in Problem [13.22] that  for any fixed (*x,y*,…,*z*).)

Thus, B = . This sum has *n*! terms composed of (*n* – 1)! terms equal to T*aa*, (*n* – 1)! terms equal to T*bb*, …, and (*n* – 1)! terms equal to T*cc*. So,

B = (*n* – 1)! (T*aa* + T*bb* + … + T*cc*) = (*n* – 1)! Tr (A) = (*n* – 1)! Tr

Similarly,

= (*n* – 1)! Tr , **… ,** = (*n* – 1)! Tr .



I believe that (\*) merits being a standalone problem all on its own.